FERMI LIQUID AND ITS APPLICATIONS

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ABSTRACT: Fermi liquids represent an intriguing state of matter that exhibits unique properties at low temperatures, making them relevant for diverse scientific fields ranging from condensed matter physics to quantum computing. The Fermi liquid theory emanated from the first phenomenological approach proposed by L.D. Landau in explaining the properties of liquid ^{3}He. These many-body systems consist of interacting fermions displaying collective behavior while retaining individual particle characteristics. The Fermi liquid theory explains various properties of fermionic systems, such as specific heat, magnetic susceptibility, and dynamical response to external fields using the fundamental aspects , including quasiparticles, Landau parameters, and transport phenomena. Furthermore, practical applications leveraging the exceptional features of Fermi liquids have emerged across several industries. For instance, Fermi liquid behaviors play a critical role in understanding unconventional superconductivity, spintronics, and heavy fermion materials. Moreover, they offer insights into designing novel electronic devices, developing highly efficient cooling methods, and building scalable quantum computers. Given promising outlook for both basic science and technological breakthroughs, this paper aims to summarize the essential concepts of Fermi liquids and discuss emerging applications driving innovation and progress in this captivating area of research.

Keywords: Applications, Fermi liquid, Landau Parameters, Quasiparticles.

Introduction

Fermi liquids (FLs) represent a conceptual model used to explain how electrons interact within a solid material that exhibit similarities to a collection of a gas of non-interacting particles at lower temperatures (Kuramoto, 2020). This behavior is explained through Fermi liquid theory, which describes the interactions among these particles (Eschrig *et al.,* 2001).

A Fermi liquid (FL) can also refer to a quantum many-particle system composed of interacting fermions, which are particles with half-integer spins, including electrons, neutrons, protons, ^{3}He atoms, and quarks. The Fermi liquid theory elucidates various properties of fermionic systems, such as specific heat, magnetic susceptibility, and their dynamical response to external fields (Kuramoto, 2020) .

The key characteristics of these fermions include a well-defined Fermi surface, which is a boundary in momentum space separating filled and empty states for fermionic particles. This surface represents the divide between occupied and unoccupied energy levels at absolute zero temperature (Das & Green, 2016).

Fermi liquids adhere to the Pauli Exclusion Principle, which dictates that no two identical fermions can simultaneously occupy the same quantum state (Kaplan, 2021). In practice, this imposes a restriction on the number of fermions allowed within a specific orbital, shell, or band.

In Fermi liquids, collective excitations are represented by quasiparticles. These quasiparticles are not fundamental particles but arise from interactions among the system's constituent particles. They possess the quantum numbers and fractional charge of the constituent particles, yet their effective masses and lifetimes differ from those of free particles (Ahn & Das Sarma, 2021). Quasiparticles can also be employed to detect topological quantum phase transitions (Manna *et al*., 2020).

The Fermi liquid theory, which explains the behavior of strongly correlated fermions, emerged from the fusion of Landau's theory of Fermi liquids and the BCS theory of superconductivity (Chen & Kou, 2011). Landau suggested that a system of strongly interacting fermions could form a "Fermi-liquid state," where low-energy properties are dominated by fermionic excitations known as quasi-particles (Eschrig *et al.*, 2001). These quasi-particles are composite states of elementary fermions with significant interactions (Kulaxizi & Parnachev, 2008). The concept of quasi-particles is fundamental to the Fermi liquid theory, enabling the description of interaction effects with a limited number of parameters (Rozali & Smyth, 2014).

Overall, Fermi liquids are crucial for understanding the behavior of strongly correlated fermions and have applications across various scientific disciplines.

Theory and Description of Fermi Liquid

The Fermi liquid theory, developed by Landau, is a powerful framework for describing strongly correlated fermions. It is based on the concept of quasi-particles and effectively captures the interaction effects in a system. The theory has been successfully applied to various phenomena, such as specific heat and magnetic susceptibility, providing a quantitative understanding of these properties (Wilmington *et al.,* 2021).

Additionally, the theory has been utilized to examine the divergences of the irreducible vertex function in dynamical mean field theory, providing insights into the breakdown of Fermi liquid theory at the Mott transition (Aldape *et al*., 2020).

At finite temperatures, the probability of finding a fermion in a specific energy state is described by the Fermi Distribution Function:

$$f(\epsilon, \mu, T) = \frac{1}{1+\exp[\beta(\epsilon-\mu)]} \qquad (1)$$

where ϵ denotes the energy level, μ symbolizes the chemical potential, and $\beta = \frac{1}{k_B T}$, with k_B standing for the Boltzmann constant and T symbolizing the temperature.

The momentum equivalent to the Fermi Energy is called the Fermi momentum p_F and is, calculated using the relationship:

$$p_F = \hbar \sqrt[3]{\frac{3\pi^2}{V} N} \qquad (2)$$

with $\hbar$ denoting Planck's constant divided by 2π, V representing the volume, and N symbolizing the total number of fermions.

Dividing the Fermi Momentum by the fermion's mass gives the Fermi Speed v_F and is calculated as:

$$v_F = \frac{p_F}{m} \qquad (3)$$

Quasiparticles and their properties

Quasiparticles lie at the heart of Fermi Liquid Theory (FLT), acting as the fundamental building blocks for understanding the behavior of weakly interacting fermions. Born from the renormalization of actual fermions, quasiparticles inherit the fermion's quantum numbers, but acquire different properties, such as mass, charge, and spin.
In FLT, the dynamics of quasiparticles translate into a vivid depiction of low-energy excitations. Akin to the propagation of ripples on a calm lake's surface, quasiparticles behave as oscillations riding atop an ocean of interacting fermions.

Quasiparticles are a subject of interest in numerous papers. Ahn and Das Sarma (2021) explored the properties of quasiparticles in a two-dimensional electron gas with long-range Coulomb interactions and inherent mass anisotropy.

Two principal energy scales define the realm of validity for FLT: The Fermi Energy (E_F), which determines the threshold for quasiparticle decay. In FLT, quasiparticles survive longer than actual fermions when

$$|E - E_F| \ll E_F \qquad (4)$$

The temperature (T) scale prescribes the dominion of quasiparticle behavior. FLT reigns supreme when $K_B T \ll E_F$

Landau's Phenomenological Approach

Building on previous work by Sommerfeld, Bethe, and Bloch, Landau sought to construct a universal framework to account for the subtle differences between non-interacting Fermi gases and realistic systems with weakly interacting fermions. His intuition gave birth to a remarkably elegant and powerful approach, christened Fermi Liquid Theory (FLT).

Landau posited that weakly interacting fermions behaved as if they were nearly independent entities, except for minuscule corrections attributed to mutual interactions. This premise implied that each fermion acquired a dressing, converting it into a heavier entity known as a quasiparticle. The quasiparticle maintained the fermion's identity, but with renormalized properties, such as mass, charge, and spin (Landau, 1959)

To formalize his vision, Landau expressed the energy of a Fermi liquid in terms of quasiparticle occupation numbers, labeled n_k. Mathematically, the total energy reads:

$$E = \sum_k n_k \in_k \qquad (5)$$

where $\in_k$ denotes the quasiparticle energy measured relative to the chemical potential (μ). The interaction between quasiparticles entered through the dependence of $\in_k$ on the occupation numbers n_k , assuming the form:

$$\delta \in_k = \sum_{k'} f_{kk'} \delta_{nk'} \qquad (6)$$

Here, $f_{kk'}$ represents the scattering amplitude, encoding the strength of quasiparticle interactions. Landau further assumed that $f_{kk'}$ remained invariant under Galilean transformations.

An essential aspect of FLT concerned the longevity of quasiparticles. Landau recognized that quasiparticles survived much longer than actual fermions, enabling a meaningful quasiparticle description. Indeed, quasiparticles persisted sufficiently long enough to engage in collisions, justifying the assumption of equilibrium thermodynamics.

The strength of quasiparticle interactions became encoded in dimensionless Landau Parameters, represented as ${F_1}^{\alpha}$ Here, α indexed conserved quantities (such as energy, momentum, angular momentum, or spin), while l indicated multipole orders.

Landau's phenomenological approach marked a turning point in the history of many-body physics. It succeeded in casting light on the puzzling features exhibited by interacting fermion systems, establishing a firm foundation for studying complex phenomena, such as superconductivity, magnetism, and transport anomalies.

Ground State Energy and Landau Parameters

The ground state energy (E_0) of a Fermi liquid reflects the aggregate energy harbored by the system when all quasiparticles settle into their lowest-energy states. In FLT, E_0 responds to infinitesimal variations in quasiparticle occupation numbers, leading to a concise formula:

$$E_0 = \sum_k n_{k^{(0)}} \varepsilon_{k^{(0)}} \tag{7}$$

where$n_{k^{(0)}}$ denotes the ground state occupation number, and $\varepsilon_{k^{(0)}}$ labels the corresponding quasiparticle energy.

Landau Parameters gauge the intensity of quasiparticle interactions within a Fermi liquid. Encoded in dimensionless constants, these quantities arise from comparing the energy shift experienced by a test quasiparticle amidst a sea of neighbors versus isolated conditions. Total energy expansions around the ground state assume the form:

$$E = E_0 + \sum_v F_{v^{(s)}}{}_{\Delta {N_v}^2} + \sum_\lambda G_{\lambda^{(\alpha)}}{}_{\Delta\, {M_\lambda}^2 + O(\Delta^3)} \tag{8}$$

Here, indices (ν) and (λ) span angular momenta channels, while (s) and (α) distinguish between scalar and axial responses. Symbols ΔN_V and ΔM_λ denote deviations from the ground state population.

Expressing the energy change (ΔE) as a bilinear function of the occupation number fluctuation (Δn_k) reveals the connection between Landau Parameters and quasiparticle interactions as:

$$\delta E_k = \sum_{kk'} f_{kk'} \delta n_k \delta_{nk'} \tag{9}$$

Imposing symmetry constraints and demanding orthonormality of Legendre polynomials yield explicit expressions for Landau Parameters in terms of quasiparticle scattering amplitudes, namely,

$$ {F_v}^{(s)} \frac{V}{(2v+1)^2 \sum \sigma} \int d\Omega\, P_v(cos\theta) f_{\sigma\sigma'}(\theta, \phi) \tag{10}$$

$$ {G_\lambda}^{(a)} = -\frac{V}{(2\lambda+1)^2 \sum \sigma\sigma'} (-1)^{\sigma+\sigma'} \int d\Omega\, P_\lambda(cos\theta) g_{\sigma\sigma'}(\theta, \phi) \tag{11}$$

where $f_{\sigma\sigma'}(\theta, \phi)$ and $g_{\sigma\sigma'}(\theta, \phi)$ stand for spin-dependent scattering amplitudes.

2.4. Effective Mass and Compressibility

Prominent Landau Parameters capture the effective mass enhancement (m^*) and the inverse compressibility (κ^{-1}). Relationships connecting these quantities to Landau Parameters read:

$$m^*/m = 1 + \frac{F_1^{(s)}}{3}, \tag{12}$$

$$\kappa^{-1} = \rho^2 \frac{\partial \mu}{\partial \rho} = \frac{1}{n}(1 + \frac{F_0^{(s)}}{3})^{-1} \tag{13}$$

Landau Parameters lose accuracy when applied to systems with vanishing densities or strong correlations. In such cases, alternate descriptions, such as the Hubbard model or dynamical mean-field theory, gain ascendency.

Applications of Fermi Liquid Theory

Fermi Liquid Theory is prominent in different areas of physics, providing answers to complex problems like superconductivity, magnetism, and heavy fermion behavior. The theory has been backed up by numerous experimental findings, which has strengthened its reputation and solidified its status as a fundamental concept in the field of quantum many-body physics.

FLT continues to shape modern physics, playing a central role in the investigation of condensed matter systems, astrophysical phenomena, and quantum technologies. Among its numerous achievements, FLT correctly predicted the superfluid phases of helium-3 and the exotic excitations seen in quantum magnets. Moreover, FLT forms the basis for comprehending high-temperature superconductivity, topological insulators, cold atoms, and relativistic plasmas. As our fascination with quantum many-body systems grows, so does the influence and relevance of Fermi Liquid Theory.

Superconductivity

Fermi liquids played a crucial role in the development of the theory of superconductivity. In 1957, John Bardeen, Leon Cooper, and John Robert Schrieffer (BCS) formulated the BCS theory, explaining superconductivity as a result of Cooper pairs of electrons formed due to electron-phonon interactions.

Superconductivity is a solid-state physics phenomenon that occurs when certain materials are cooled below a critical temperature, known as Tc. Superconductors exhibit zero electrical resistance and no indoor magnetic field. (Shan, 2019; Kagan, 2013)

Fermi liquid concepts, such as the existence of a well-defined Fermi surface and quasiparticles, are essential for describing the behavior of electrons in superconductors. The stability of the Fermi surface and the presence of quasiparticles contribute to the pairing and transport properties that enable the flow of supercurrent without resistance.

Heavy fermion materials

In the hallowed halls of condensed matter physics, Heavy Fermion Compounds challenged the authority of Fermi Liquid Theory, as their colossal effective masses and staggering specific heat coefficients defied explanation. Understanding their behavior required extending the concept of Fermi liquids to include strongly correlated electron systems, leading to the development of theories such as the Kondo effect and the Anderson lattice model

Heavy fermion materials are strongly correlated electron systems that exhibit diverse quantum ground states such as antiferromagnetic order, ferromagnetic order, non-Fermi-liquid phases, unconventional superconductivity, quantum spin liquids, orbital order, and topological order (Wu *et al.*, 2019)

These materials have relatively small characteristic energy scales, allowing different quantum states to be tuned continuously by external parameters such as pressure, magnetic field, and chemical doping (Vaňo *et al.*, 2021).

High-temperature superconductivity

The study of high-temperature superconductors in the 1980s and 1990s further underscored the importance of Fermi liquid concepts. These materials exhibit unconventional superconducting properties and are not easily explained by traditional BCS theory. High-temperature superconductivity refers to the phenomenon of materials exhibiting superconductivity at temperatures higher than the boiling point of liquid nitrogen (-196°C). Despite decades of research, a comprehensive microscopic theory of high-temperature superconductivity is still lacking (Zhou *et al.*, 2021).

High-temperature superconductors have potential applications in high-field magnets, low-loss transmission cables, and superconducting magnetic energy storage due to their high critical magnetic field and critical temperature (Hongjun *et al*., 2018).

Quantum Hall Effect

The quantum Hall effect is an unconventional phenomenon where a voltage can be generated by two perpendicular currents. It can survive under time-reversal symmetry and is sensitive to the breaking of discrete and crystal symmetries. It is a quantum transport phenomenon that has a deep connection with the Berry curvature. A full quantum description of the quantum Hall effect is still absent. However, recent research has made progress in constructing a quantum theory of the nonlinear Hall effect using diagrammatic techniques. These techniques have shown qualitative and quantitative differences compared to the semiclassical Boltzmann formulism (Du *et al*., 2021).

Spintronics and quantum computing

Fermi liquid behavior in materials can impact spin transport and spin dynamics, which are crucial for spintronic devices and quantum computing. Further exploration of Fermi liquid properties and their control in materials may lead to advancements in spin-based technologies and the development of novel quantum computing platforms. Embodying the apotheosis of modern computational prowess, quantum computers stride boldly into terra incognita, navigating treacherous waters strewn with pitfalls and promises alike. At the very core of this brave new world lies Fermi Liquid Theory (FLT), quietly weaving the threads of order amidst the chaos. In addition, the Fermi energy theory is applied in spintronics to understand spin-to-charge conversion and spin transport phenomena. In the rarefied atmosphere of topological quantum computation, the scientists braid the exotic quasiparticles of the quantum realm, weaving a tapestry of entanglement that

Energy-related applications

Understanding Fermi liquid behavior in materials relevant to energy conversion and storage, such as thermoelectrics and catalysts, can contribute to the development of efficient energy technologies. Exploring the impact of Fermi liquid properties on electronic and thermal transport in these materials may lead to improved performance and energy sustainability. The Fermi energy theory has various applications in energy-related fields. It can be used to study the spectral structure of real gases, such as the hard-sphere gas, gas in porous media, and gas in an external potential field or curved background (Pang & Wu, 2016).

Materials Design and Engineering

Fermi energy theory is applied in materials design and engineering to optimize the thermoelectric properties of compounds and to study the electronic and phonon structure of advanced thermoelectric materials (Kawamura *et al*., 2021).

Experimental Techniques

A plethora of experimental tools exists for investigating Fermi liquids, spanning from bulk probes like heat capacity and magnetization to microscopic imaging techniques like scanning tunneling microscopy. Angle-resolved photoemission spectroscopy offers detailed insights into band structures and quasiparticle lifetimes, while optical spectroscopy reveals information about plasmons and single-particle excitations. New developments, such as time- and angle-resolved photoemission spectroscopy, allow us to probe nonequilibrium dynamics on femtosecond timescales.

Conclusion

Continued research in Fermi liquid theory, coupled with the exploration of new materials and phenomena, holds the potential for exciting discoveries, the development of novel technologies, and the advancement of our understanding of quantum many-body systems. The interdisciplinary nature of Fermi liquid research ensures its relevance in fields ranging from condensed matter physics to materials science, quantum information, and beyond.

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